Discrete Mathematics

The Backbone of Mathematics and Computer Science

Sets WorkBook

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1 List the elements in the following sets:
   a. {x | x is a real number such that $x^2 = 4$}
   b. {x | x is a positive integer less than 8}
   c. {x | x is the largest positive integer whose square is less than 100}

2 Suppose that A = {2, 4, 6, 8}, B = {4, 6}, C = {4, 6, 8}. Determine which of the sets are subsets of which other of these sets.

3 For the following sets, determine if 1 is an element of that set.
   a. {x ∈ **R** | x is an integer greater or equal to 1}
   b. {1, 1}
   c. {1, {1}}
   d. {{1}, {1, {1}}}

4 Determine whether each of the following is true or false.
   a. 0 ∈ ø
   b. ø ∈ {0}
   c. {ø} ⊆ {0}
   d. {ø} ⊆ {ø}
   e. ø ∈ {0, ø}

5 Use set builder notation to describe the given sets:
   a. {0, 2, 4, 6, 8}
   b. {1/2, 1/4, 1/6, 1/8, 1/10, …}
   c. {-2, -1, 1, 2}

6 How many elements are in the set {5, {5}, {5, 5}, {{5}}}?

7 Let A = {x ∈ Z | x = 5a for some integer a}, B = {y ∈ Z | y = 2b -1 for some integer b}, and C = {z ∈ Z | z = c + 4 for some integer c}. Clarify which set is a subset of which other set.

8 What is the cardinality of each of these sets?
   a. ø
   b. {ø, ø}
   c. {ø, {ø, {ø}}}
   d. *P*(*P*(ø))

9 Find the power set of each of these sets:
   a. {1}
   b. {1, 2}
   c. {1, {1}}

10 Determine whether each of these sets is the power set of a set.
   a. ø
   b. {ø, {1}, {1, ø}}
   c. {ø, {1}}

11 Let A = {1, 2, 3}, B = {x, y}, and C = {0, 1}. Find:
   a. A x B
   b. B x C
   c. A x B x C

12 Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$. Find:
   a. $A \cup B$
   b. $A \cap B$
   c. $A - B$
   d. $B - A$

13 Find the set A and B if $A - B = \{1, 7, 8\}$, $B - A = \{2, 4\}$, and $A \cap B = \{3, 6, 9\}$.

14 Draw the Venn Diagrams for the following set combinations:
   a. $A \cap (B \cup C)$
   b. $A \cap B \cap C$
   c. $(A - B) \cup (A - C) \cup (B - C)$

15 Let the universal set be the Real Numbers. Let $A = \{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$, $B = \{x \in \mathbf{R} \mid -3 < x < 2\}$, and $C = \{x \in \mathbf{R} \mid 0 < x \leq 4\}$. Find each of the following:
   a. $A \cup B$
   b. $A \cap B$
   c. A^c
   d. $(A \cup C)^c$
   e. $A^c \cap B$

16 Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$, and $C = \{4, 5, 6, 7, 8, 9, 10, 11\}$. Find:
   a. $(A \cup B) \cap C$
   b. $(A \cap B) \cup C$
   c. $A \cap B \cap C$

17 Draw the Venn Diagrams for the following set combination:
   a. $A - (B - C)$
   b. $(A \cap B) \cup (A \cap C)$

18 Let A, B and C be sets. Using Venn Diagrams, show if the following is equal or not. $(A - B) - C \overset{?}{=} (A - C) - (B - C)$.

19 Complete the following sentence without suing the symbols ∩**,** ∪ or -.
   a. $x \notin A \cup B$ if, and only if,______.
   b. $x \notin A \cap B$ if, and only if,______.
   c. $x \notin A - B$ if, and only if,______.

20 Consider the following collections of subsets of $R = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. State which one is a partition of S:
   a. [{0, 1, 3, 5, 7}, {4, 6, 8, 9}]
   b. [{0, 1}, {2, 3, 4, 5}, {5, 6, 7, 8, 9}]
   c. [{0}, {1, 2, 3}, {4, 5}, {6, 7, 8, 9}]

21 Which of the followings are an example of the empty set:
   a. The set of odd natural numbers divisible by 2
   b. $\{x : x \in N, 9 < x < 10\}$
   c. $A = \{\varnothing\}$

22 State which of the followings are true and which are false:
   a. $7{,}747 \in \{x \mid x \text{ is a multiple of } 37\}$
   b. $A \cup \varnothing = \{A, \varnothing\}$

23 Given that A = {1, 2, 3, 4, 5}. If x represents any member of A, then find the following sets containing all the numbers represented by:
   a. $x + 1$
   b. x^2

24 Let A = {a | a can be divided by 2 with no remainder}, and B = {b | b can be divided by 3 with no remainder}. Then determine if the following is true or false. $A \cap B = ø$.

25 For all sets, A, B and C, is $(A – B) \cap (C – B) = (A \cap C) – B$?

26 Are $\mathbb{N}$ and $\mathbb{Z}$ disjoint?

27 Show if the following statement is equal or not. $(A \cap B`) \cup (A \cap B) \stackrel{?}{=} A$. Use Venn Diagrams.

28 Is {{1, 5}, {4, 7}, {2, 8, 6, 3}} a partition of {1,2,3,4,5,6,7,8}?

29 Let E be the set of the even integers and O the set of the odd integers. Is {E, O} a partition of Z (the set of all integers)?

30 Let A = {1, 2} and B = {2, 3}. What is P(A U B), where P is the power set?

ANSWERS

1 List the elements in the following sets:

a. {x | x is a real number such that $x^2 = 4$}

The square of 2 and -2 is 4. Hence the elements of the set { x | x is a real number such that $x^2 = 4$} is {2, -2}.

b. {x | x is a positive integer less than 8}

The list of positive integers = {1, 2, 3, 4, …}.
The positive integers less than 8 = 1, 2, 3, 4, 5, 6 and 7.
Hence the elements of the set { x | x is a positive integer less than 8} is {1, 2, 3, 4, 5, 6, 7}.

c. {x | is the largest positive integer whose square is less than 100}

The square of 10 is 100. The positive integer less than 10 is 9. The square of 9 is 81. Hence the elements of the set { x | x is a positive integer whose square is less than 100} is {9}.

2 Suppose that A = {2, 4, 6, 8}, B = {4, 6}, C = {4, 6, 8}. Determine which of the sets are subsets of which other of these sets.

Set Q is a subset of set P if all the elements of set Q is in set P, and every set is a subset of itself.
Hence, $B \subseteq A$, $B \subseteq C$, and $C \subseteq A$.

3 For the following sets, determine if 1 is an element of that set.

a. {$x \in$ **R** | x is an integer greater or equal to 1}

The set {$x \in$ **R** | x is an integer greater or equal to 1} is the set that contains all the integer that are greater or equal to 1. Meaning {1, 2, 3…}. Thus 1 is an element of {$x \in$ **R** | x is an integer greater or equal to 1}.

b. {1, 1}

1 is an element of the set {1, 1}.

c. {1, {1}}

The set {1, {1}} has two elements: 1, and {1}. The second element is a set in itself. Hence 1 is an element of {1, {1}}.

d. {{1}, {1, {1}}}

The set {{1}, {1, {1}}} has two elements: {1}, and {1, {1}}. Both these elements are sets and non of these elements are equal to 1. Therefore 1 is not an element of the set {{1}, {1, {1}}}.

4 Determine whether each of the following is true or false.

a. $0 \in \emptyset$

False. The empty set has no elements, hence 0 can't be an element of the empty set.

b. $\emptyset \in \{0\}$

False. The empty set is a subset of every set but one can't use "an element of" notation instead of "a subset of". Thus the element ø doesn't belong to the set {0}.

c. $\{\emptyset\} \subseteq \{0\}$

False. $\{\emptyset\} = \{\{\}\}$. The set {ø} is a set that has one element which is an empty set; however the set {0} doesn't have an element that is a set that has an empty set inside it. Hence {ø} can't be a subset of {0}.

d. $\{\emptyset\} \subseteq \{\emptyset\}$

True. Every set is a subset of itself hence the set containing the empty set is a subset of itself.

e. $\emptyset \in \{0, \emptyset\}$

True. The left hand side is a set that contains 0, and the empty set. Hence the empty set is an element of the set {0, ø}.

5 Use set builder notation to describe the given sets:

a. {0, 2, 4, 6, 8}

All the elements in the set are on the form of 2n such that n is an integer between 0 - 4 including both 0 and 4. Hence the set builder notation for {0, 2, 4, 6, 8} is $\{2n \mid n \in \mathbf{Z}, 0 \leq n \leq 4\}$.

b. {1/2, 1/4, 1/6, 1/8, 1/10, …}

All the elements in the set are on the form of 1/2n such that n is an integer between 1 and ∞ including 1 but not ∞. Hence the set builder notation for {1/2, 1/4, 1/6, 1/8, 1/10, …} is $\{1/2n \mid n \in \mathbf{Z}, 1 \leq n < \infty\}$.

c. {-2, -1, 1, 2}

The elements in the set are integers between -2 and 2 including both -2 and 2, but not including 0. Hence the set builder notation for {-2, -1, 1, 2} is $\{n \mid n \in \mathbf{Z}, -2 \leq n \leq 2, n \neq 0\}$.

6 How many elements are in the set {5, {5}, {5, 5}, {{5}}}?

The set {5, {5}, {5, 5}, {{5}}} has 4 elements. They are: 5, {5}, {5, 5}, {{5}}

7 Let A = {$x \in Z$ | $x = 5a$ for some integer a}, B = {$y \in Z$ | $y = 2b - 1$ for some integer b}, and C = {$z \in Z$ | $z = c + 4$ for some integer c}. Clarify which set is a subset of which other set.

The set {$x \in Z$ | $x = 5a$ for some integer a} is all the integers that are on the from of 5a such that a is also an integer. Therefore, set A contains all the multiples of 5, meaning {…, -15, -10, -5, 0, 5, 10, 15, …}.

The set {$y \in Z$ | $y = 2b - 1$ for some integer b} contains all the odd integers .

The set {$z \in Z$ | $z = c + 4$ for some integer c} contains all the integers since an integer + 4 (another integer) will create another integer. Using this formula, you can create all the integers.

Hence, $A \subseteq C$, $B \subseteq C$.

8 What is the cardinality of each of these sets?

a. ø

The number of distinct elements of a set is called cardinality of the set. ø, is a set with 0 elements, hence its cardinality is 0.

b. {ø, ø}

Event though the set {ø, ø} seems to have 2 elements, but based on the definition, the cardinality of the set {ø, ø} is 1 because it only has 1 distinct element.

c. {ø, {ø, {ø}}}

The cardinality of the set {ø, {ø, {ø}}} is 2 because it has two elements and they are ø and a set which is {ø, {ø}}.

d. *P*(*P*(ø))

The empty set has 0 elements; therefore *P*(ø) = $2^0 = 1$.

Hence the power set of a set with one element is *P*(1) = $2^1 = 2$.

Therefore the number of the elements that are in the set *P*(*P*(ø)) is 2.

Another way to think about the problem is this: the power set of the empty set is the set, A, that contains the set it self, ø, and the empty set.

A = {ø, ø} = {ø}

P({ø}) = {{ø}, ø}. Hence the cardiaiality is 2.

9 Find the power set of each of these sets:

a. {1}

The power set of any set is the set of all the subsets, including the empty set and the set itself. Hence the power set of {1} is {{1}, ø}.

b. {1, 2}

The subsets of the set {1, 2} is {1}, {2}, {1, 2} and ø. Hence the power set of the set {1, 2} is {{1}, {2}, {1, 2}, ø}.

c. {1, {1}}

The subsets of the set {1, {1}} is {1}, {{1}}, {1, {1}} and ø. Hence the power set of the set {1, {1}} is {{1}, {{1}}, {1, {1}}, ø}.

10 Determine whether each of these sets is the power set of a set.

a. ø

The power set of a set contains subsets; however ø is not a set containing subsets. Hence ø is not the power set of any set.

b. {ø, {1}, {1, ø}}

The number of subsets in a powers is 2^n where n is the number of distinct elements in the original set. Based on this, a power set will have 1, 2, 4, 8, 16, 32, … elements. The set {ø, {1}, {1, ø}} has 3 elements; therefore it can't be the power set of any set.

c. {ø, {1}}

The subsets of the set {1} is {1} and ø. Hence {ø, {1}} is the power set of the set {1}.

11 Let A = {1, 2, 3}, B = {x, y}, and C = {0, 1}. Find:

a. A x B

A x B = {(1,x), (1, y), (2, x), (2, y), (3, x), (3, y)}.

b. B x C

B x C = {(x, 0), (x, 1), (y, 0), (y, 1)}.

c. A x B x C

A x B x C = {(1, x, 0), (1, y, 0), (2, x, 0), (2, y, 0), (3, x, 0), (3, y, 0), (1, x, 1), (1, y, 1), (2, x, 1), (2, y, 1), (3, x, 1), (3, y, 1)}.

12 Let A = {1, 2, 3, 4} and B = {2, 4, 6}. Find:

a. A U B

The union of two sets is the set that contains all the elements in both sets. Hence, A U B = {1, 2, 3, 4, 6}.

b. A ∩ B

The intersection of two sets is the set that contains all the elements that are in both sets. Hence, A ∩ B = {2, 4}.

c. A - B

The difference between set A and set B is the set that contains all the elements that are in A but not in B. Hence, A - B = {1, 3}.

d. B - A

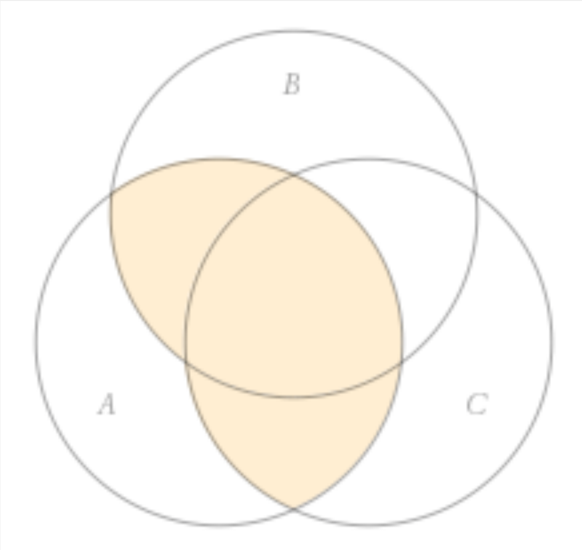
B - A = {6}

13 Find the set A and B if A - B = {1, 7, 8}, B - A = {2, 4}, and A ∩ B = {3, 6, 9}.

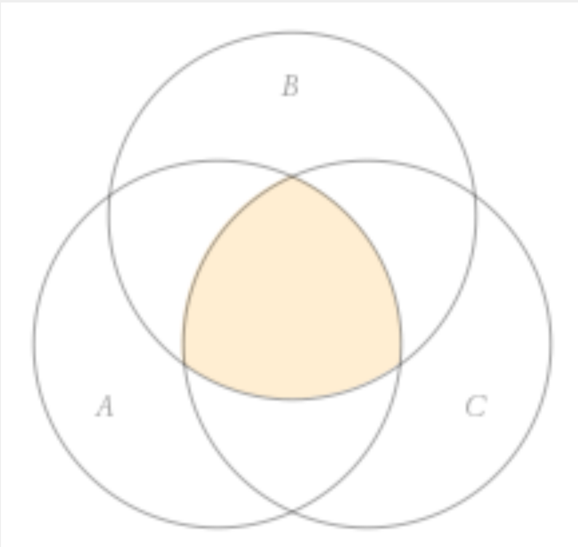
If A - B = {1, 7, 8}, then we know that A at least has 1, 7 and 8. We Also know that A ∩ B = {3, 6, 9}, which means 3, 6 and 9 are both in A and B. Hence A = {1, 3, 6, 7, 8, 9}.
B - A = {2, 4}. Hence B = {2, 3, 4, 6, 9}.

14 Draw the Venn Diagrams for the following set combinations:

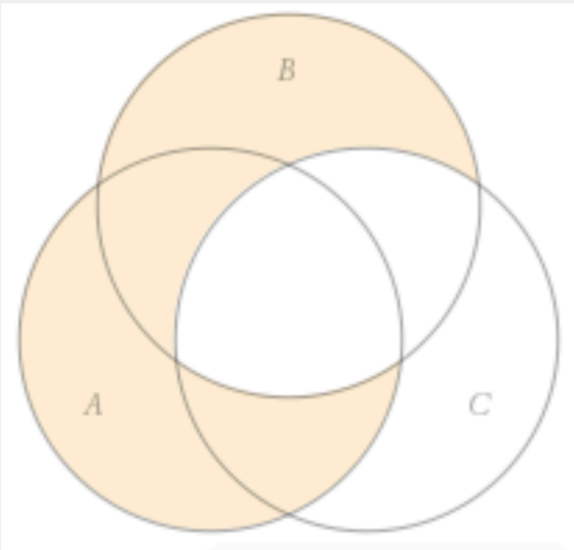
a. A ∩ (B U C)



b. A ∩ B ∩ C



c. (A - B) U (A - C) U (B - C)



15 Let the universal set be the Real Numbers. Let A = $\{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$, B = $\{x \in \mathbf{R} \mid -3 < x < 2\}$, and C = $\{x \in \mathbf{R} \mid 0 < x \leq 4\}$. Find each of the following:

a. A U B

The Universal Set is set to be all the Real Numbers. Set A contains all the Real Numbers that are great or equal to -1 and smaller or equal to 0. Set B contains all the Real Numbers that are greater than -3 and less than 2. The set A U B = $\{x \in \mathbf{R} \mid -3 < x < 2\}$

b. A ∩ B

The set A ∩ B contains all the Real Numbers that are in the set A and B. Hence the set A ∩ B = $\{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$.

c. A^c

A complement is the set that contains all the Real Numbers except the elements that are in A. Hence $A^c = \{x \in \mathbf{R} \mid -1 > x \text{ or } x > 0\}$.

d. $(A \cup C)^c$

$A \cup C = \{x \in \mathbf{R} \mid -1 \leq x \leq 4\}$. Hence, $(A \cup C)^c = \{x \in \mathbf{R} \mid -1 > x \text{ or } x > 4\}$

e. $A^c \cap B$

$A^c = \{x \in \mathbf{R} \mid -1 > x \text{ or } x > 0\}$

$B = \{x \in \mathbf{R} \mid -3 < x < 2\} = \{x \in \mathbf{R} \mid -3 < x \text{ and } x < 2\}$

Hence $A^c \cap B = \{x \in \mathbf{R} \mid 0 < x < 2, \text{ or } -3 < x < -1\}$

16 Let A = {0, 2, 4, 6, 8,10}, B = {0, 1, 2, 3, 4, 5, 6, 7}, and C = {4, 5, 6, 7, 8, 9, 10,11}. Find:

a. (A U B) ∩ C

A U B = {0, 1, 2, 3, 4, 5, 6, 7, 8, 10}. Hence (A U B) ∩ C = {4, 5, 6, 7, 8, 10}

b. (A ∩ B) U C

A ∩ B = {0, 2, 4, 6}. Hence, (A ∩ B) U C = {0, 2, 4, 5, 6, 7, 8, 9, 10, 11}

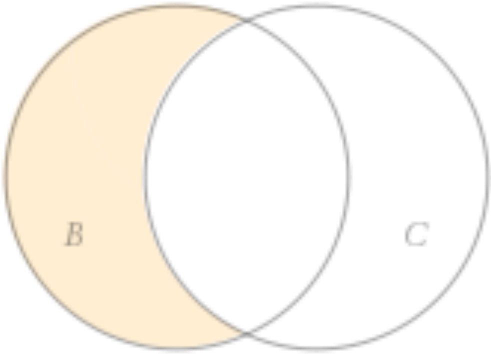
c. A ∩ B ∩ C

A ∩ B ∩ C = {4, 6}.

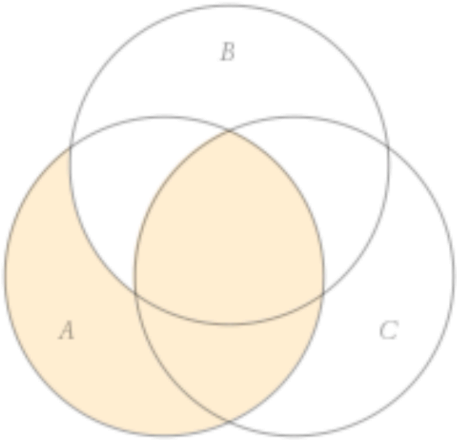
17 Draw the Venn Diagrams for the following set combinations:

a. A − (B − C)

B - C = all those elements that are in B but not in C, as shown bellow.

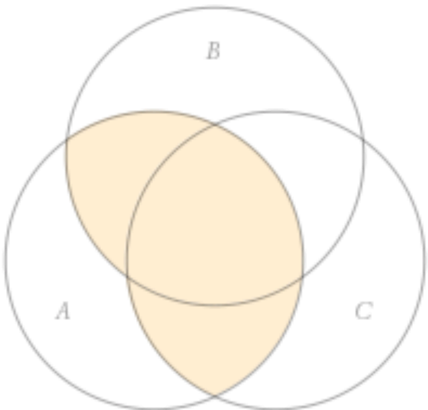


Hence A − (B − C) = all those elements that are in A but not in (B − C), as shown bellow.



b. (A ∩ B) U (A ∩ C)

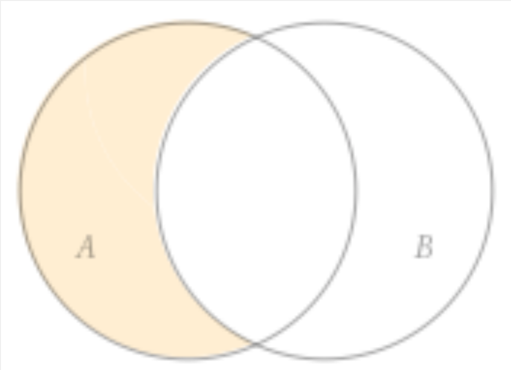
(A ∩ B) U (A ∩ C) is the set that contains all the elements that are in (A ∩ B) or (A ∩ C), as shown bellow. The previous statement wouldn't be true if it was "*…is the set that contains all the elements that are in (A ∩ B)* ***and*** *(A ∩ C)*", since that implies intersection rather than union.



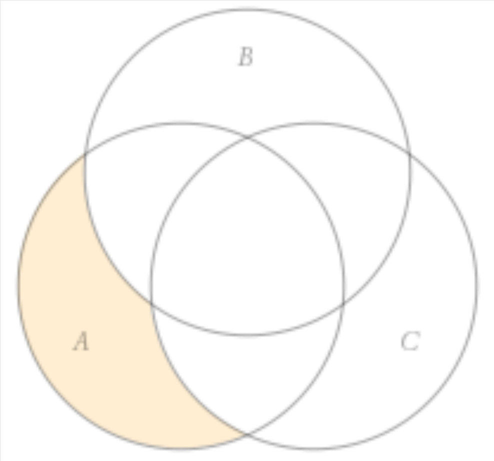
18 Let A, B and C be sets. Using Venn Diagrams, show if the following is equal or not. (A - B) - C =? (A - C) - (B - C).

For the left hand side:

A - B is the set that contains all the elements that are in A but not in B.

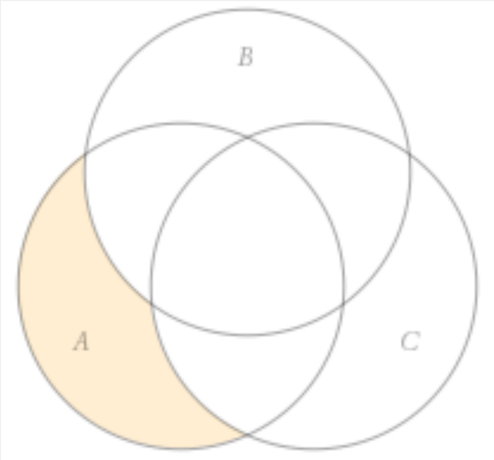


(A - B) - C is the set that contains all the elements that are in (A - B) but not in C.



For the right hand side:

(A - B) - (B - C) is the set that contains all the elements that are in (A - B) but not in (B - C).



Therefore the right hand side equals the left hand side.

19 Complete the following sentence without suing the symbols ∩, U or -.

a. x ∉ A U B if, and only if,_____.

x ∉ A U B means x is not in the union of A and B. Hence the completed sentence is: x ∉ A U B if, and only if, x ∉ A and x ∉ B.

b. x ∉ A ∩ B if, and only if,_____.

x ∉ A ∩ B means x is not in the intersection of A and B. Hence the completed sentence is: x ∉ A ∩ B if, and only if, x ∉ A or x ∉ B.

c. x ∉ A - B if, and only if,_____.

x ∈ A - B means x is in A but not in B. x ∉ A - B means x is not in A but it is in B. Hence the completed sentence is: x ∉ A - B if, and only if, x ∉ A, or x ∈ B.

20 Consider the following collections of subsets of R = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. State which one is a partition of R:

a. [{0, 1, 3, 5, 7}, {4, 6, 8, 9}]

A finite or infinite collection of nonempty sets {A1, A2, A3 ...} is a partition of a set A if, and only if: A is the union of all the subsets, and the subsets are disjoint.

(a) is not a partition of R since 2 in R doesn't belong to any of the subsets.

b. [{0, 1}, {2, 3, 4, 5}, {5, 6, 7, 8, 9}]

(b) is not a partition of R since {2, 3, 4, 5} and {5, 6, 7, 8, 9} are not disjoint.

c. [{0}, {1, 2, 3}, {4, 5}, {6, 7, 8, 9}]

(c) is a partition of R based on the definition above.

21 Which of the followings are an example of the empty set:

a. The set of odd natural numbers divisible by 2

(a) is an example of the empty set since non of the odd natural numbers can be divided by 2.

b. {x : x ∈ N, 9 < x < 10}

{x : x ∈ N, 9 < x < 10} is an example of the empty set since {x : x ∈ N, 9 < x < 10} is the set of all the natural number that are greater than 9 and less than 10, but no natural number is greater than 9 and less than 10.

c. A = {ø}

A = {ø} is NOT an example of the empty set since A = {ø} is a set that has an element which is ø.

22 State which of the followings are true and which are false:

a. 7,747 ∈ {x | x is a multiple of 37}

7,747 ∈ {x | x is a multiple of 37} is false since 7,747 is not a multiple of 37.

b. A U ø = {A, ø}

A U ø = {A, ø} is false since the union between a set and the empty set is the set itself.

23 Given that A = {1, 2, 3, 4, 5}. If x represents any member of A, then find the following sets containing all the numbers represented by:

a. x + 1

Let B = {b | b = x + 1, x ∈ A}. Thus for x = 1, b = 2, x = 5, b = 6. Hence B = {2, 3, 4, 5, 6}.

b. x^2

Let B = {b | b = x^2, x ∈ A}. Thus for x = 2, b = 4, x = 5, b = 25. Hence B = {1, 4, 9, 16, 25}

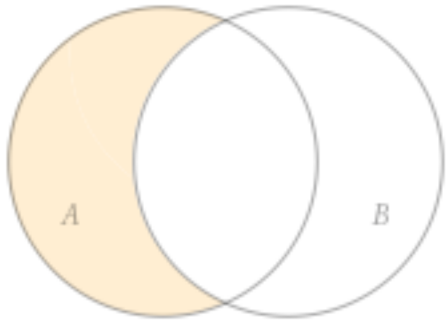
24 Let A = {a | a can be divided by 2 with no remainder}, and B = {b | b can be divided by 3 with no remainder}. Then determine if the following is true or false. A ∩ B = ø.
False. A is the set of all those numbers that can be divided by 2, thus A ={2, 4, 6, 8, …}. B is the set of all those numbers that can be divided by 3, thus B = {3, 6, 9, 12, …}. The Intersection between A and B is not the empty set, but at least it has on element and that is 6.

25 For all sets, A, B and C, is (A – B) ∩ (C – B) = (A ∩ C) – B?
Yes. You can use Venn Diagrams to show that the left hand side equals to the right hand side. But suppose:

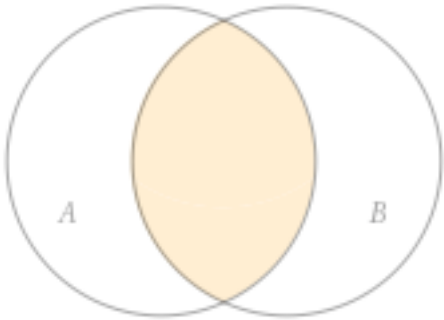
if x ∈ (A – B) ∩ (C – B), then x ∈ (A – B) and x ∈ (C – B). This means x ∈ A and x ∈ C, but x ∉ B. If x is in both A and C but not B, then x ∈ A ∩ C and x ∉ B. Meaning x ∈ (A ∩ C) – B.

26 Are **N** and **Z** disjoint?
No. By definition, two sets are disjoint if their intersection is the empty set. The intersection of the set of the Natural Numbers and the Integers is not the empty set. For example, 1 is in both **N** and **Z**.

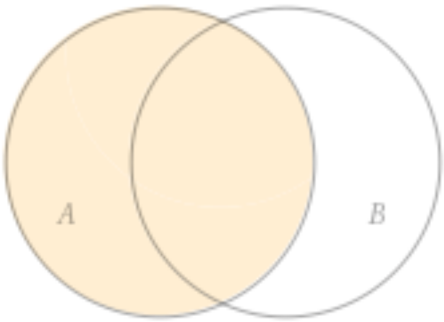
27 Show if the following statement is equal or not. (A ∩ B`) U (A ∩ B) =? A. Use Venn Diagrams.
For the set A and B, B` has all those elements that are in A but not in B, as shown bellow.



The Venn Diagram for A ∩ B is as bellow.



The Venn Diagram for (A ∩ B`) U (A ∩ B) is as bellow and it equals to A. Hence the left hand side equals to the right hand side.



28 Is {{1, 5}, {4, 7}, {2, 8, 6, 3}} a partition of {1,2,3,4,5,6,7,8}?
By definition, the set {{1, 5}, {4, 7}, {2, 8, 6, 3}} is a portion of the set {1,2,3,4,5,6,7,8}.

29 Let E be the set of the even integers and O the set of the odd integers. Is {E, O} a partition of **Z** (the set of all integers)?
No even integer is odd. The intersection of E and O = ø. The Union of E and O = **Z**. Hence by definition, {E, O} is a partition of **Z**.

30 Let A = {1, 2} and B = {2, 3}. What is P(A U B), where P is the power set of a set?
A U B = {1, 2, 3}. P(A U B) is the set of all the subsets of {1, 2, 3}.
Hence P(A U B) = {ø, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}